Two-Asset American-Style Maximum Call: Early-Exercise Structure

In the standard situation of a vanilla American-style call, the stopping area is two-dimensional in the (S,t)-plane, and the bounding curve $S_f(t)$ for $t \to T$ ends at the limiting point

$$S = \max\{K, \frac{r}{\delta}K\}\,. \qquad (*)$$

In the three-dimensional situation of the (S_1, S_2, t)-space, the stopping area consists of one or more *solids*, which are bounded by one or more *surfaces*.

For a schematic visualization of a general scenario of a two-asset option see Figure 1. This figure sketches three versions of the (S_1, S_2)-plane, in which the asset prices move for $0 \le t \le T$. Price paths start at $(S_1(0), S_2(0))$, and may reach the surface of a stopping area (shown in red).

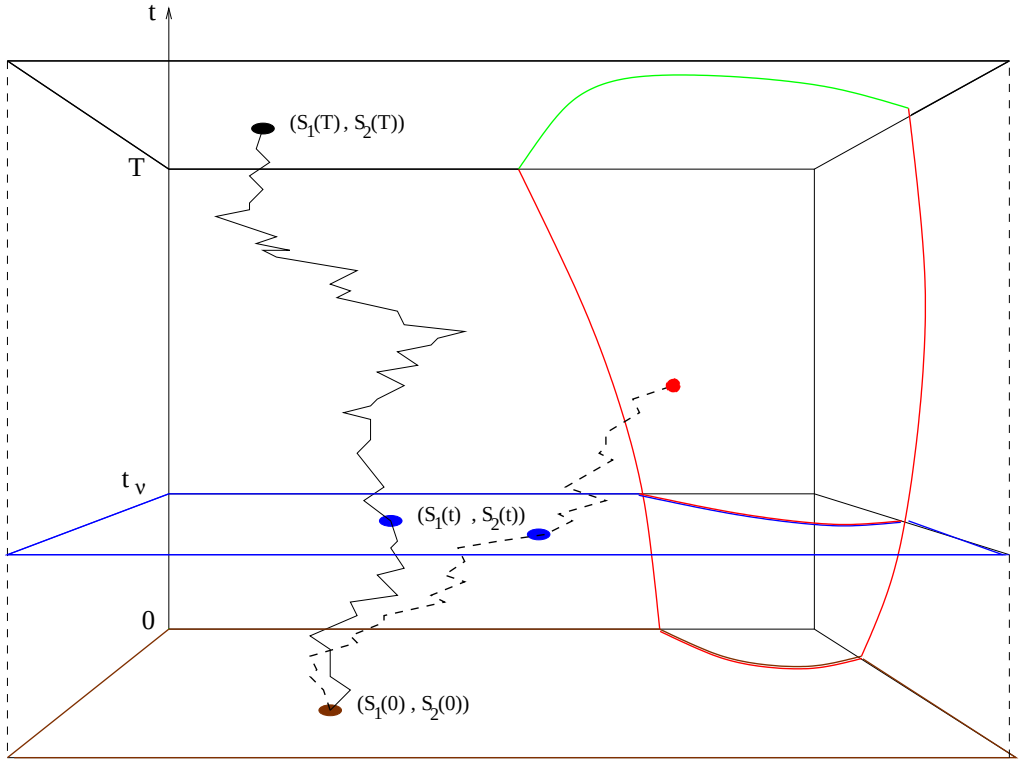

Figure 1. (S_1, S_2, t)-space with early-exercise structure of a two-asset option, schematically.

Example. We choose a max call, with payoff $\Psi(S_1, S_2) = (\max(S_1, S_2) - K)^+$ as shown in Figure 6.2 of [Seydel: Tools fCF]. Figure 2 shows cross sections through the stopping areas. For a max call, these areas consist of two parts, separated by the line $S_1 = S_2$. For each S-axis, the limiting values of $(*)$ for $t \to T$ hold also in the two-asset case, but the connecting limiting *curve* in 2D can be more fancy. In the left-hand column of two cross sections we choose a dividend rate δ_1 with $r/\delta_1 < 1$, and for the right column $r/\delta_1 > 1$ holds. The limiting curve (for $t \to T$) is shown in green/red. All cross sections were calculated with a tree method with $M = 200$, starting at $(S_1, S_2) = (1, 1)$. Parameters: $K = T = 1$, $r = 0.1$, $\delta_1 = 0.2$ or 0.05, $\delta_2 = 0.3$, $\sigma_1 = 0.2$, $\sigma_2 = 0.3.$, $\rho = 0.25$.

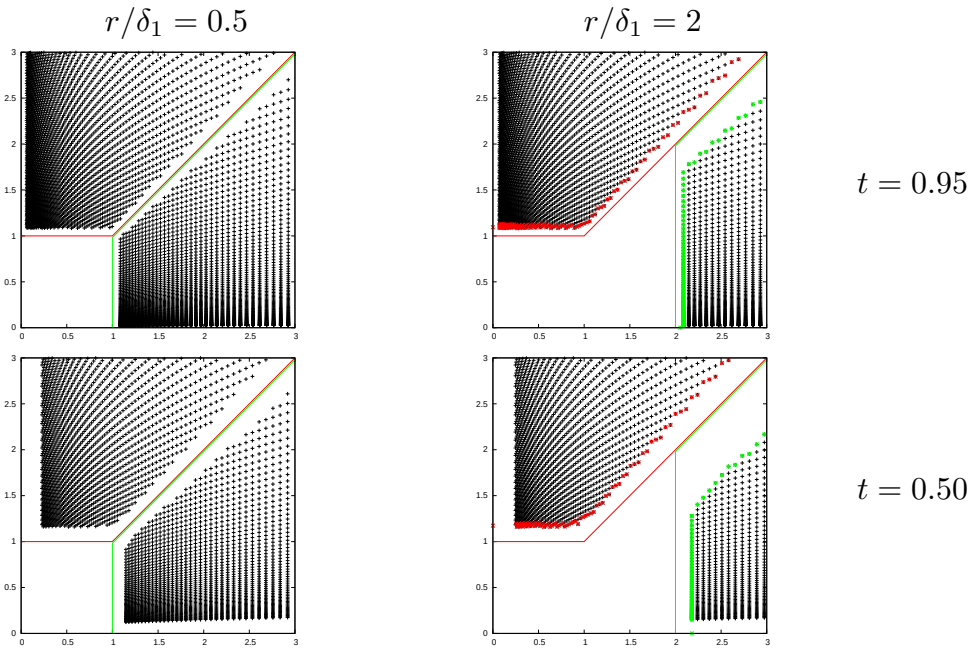

Figure 2. (S_1, S_2)-cross sections through the early-exercise area, for $t = 0.95$ and 0.50, approximated by nodes of a tree method. black: part of the stopping areas. red and green points: approximations of its boundary (early-exercise)

Even with the sparse resolution and the limited extension of our tree, we can visualize the early-exercise structure of the example. Figure 3 matches Figure 1 and the right-hand plots of Figure 2. There are two early-exercise surfaces, one indicated in red, and one in green. For $t \to T = 1$ the surfaces end in the lines shown in Figure 2 (right) and Figure 3. The continuation area is the solid in the gap between the red and the green surface.

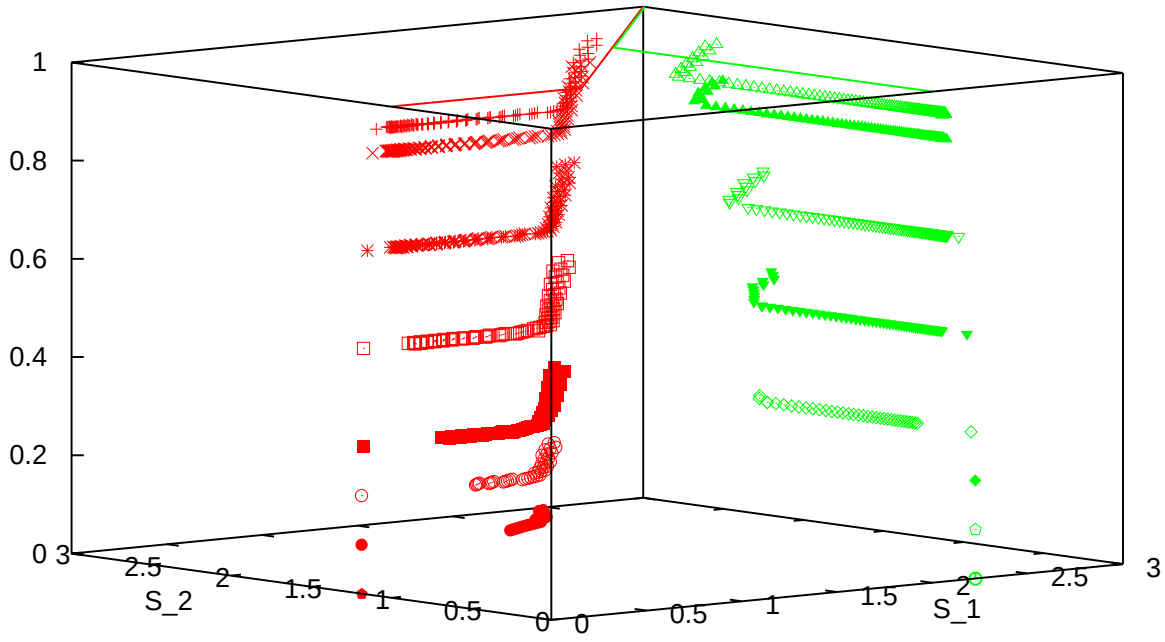

Figure 3. Cross sections in (S_1, S_2, t)-space, for $t = 0.1, 0.2, 0.3, 0.5, 0.7, 0.9, 0.95$; $r/\delta_1 = 2$. The two lowest curves of the green surface are missing because they are not touched by the tree. End points at the face planes $S_1 = 0$ and $S_2 = 0$ were calculated separately.