Discrete Dividend I: Non-Connected Stopping Region

Example: American-style put with strike $K = 3$, $T = 0.5$, $S_0 = 2.4$, $r = 0.08$, $\sigma = 0.4$, with one dividend $D = 0.06$ paid at time $t_D = 0.3$. For these parameters* the stopping region consists of two parts, the upper part with $t \geq t_D$, and the lower part with $t \leq \tilde{t}$,

$$\tilde{t} = 0.052467$$

(see Exercise 4.1b). For reference, the price calculated with a binomial method and mesh fineness $M = 1000$ is $V(S_0, 0) = 0.66538$.

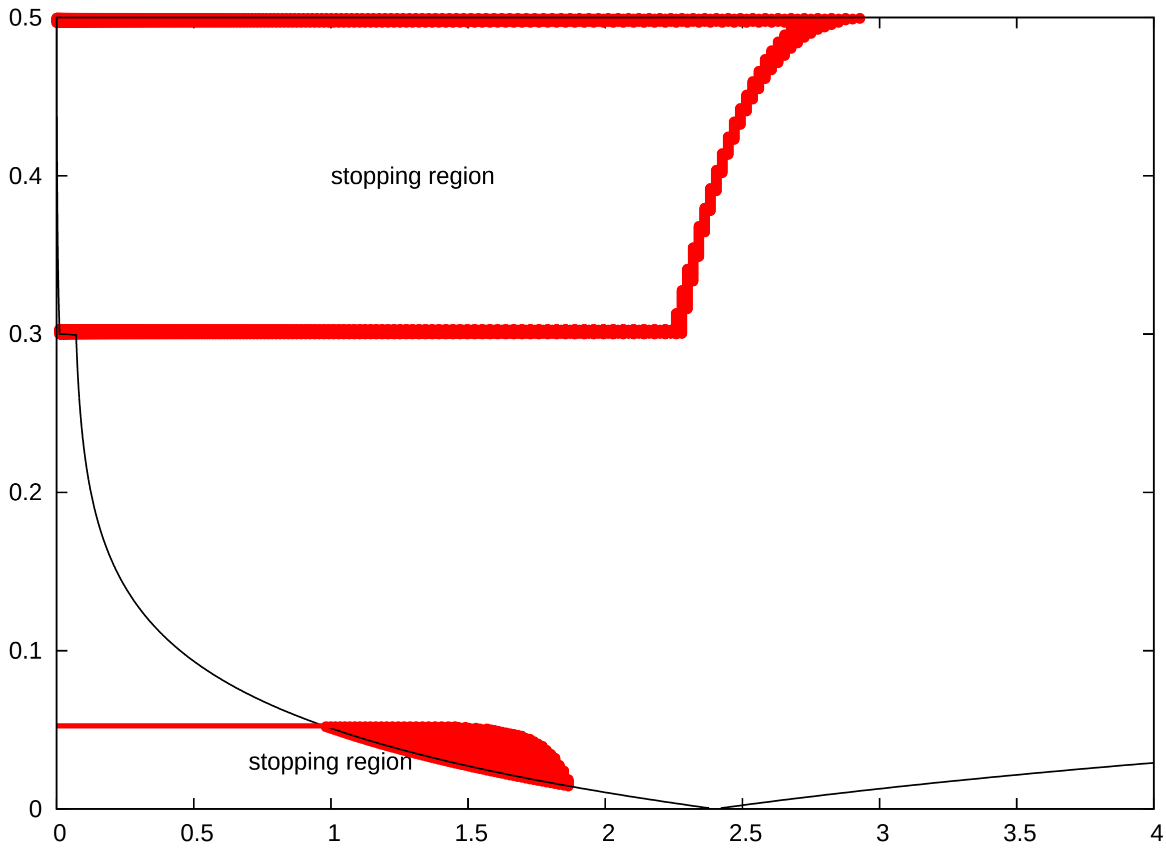

Figure 1: horizontal axis: S, vertical axis: t.
Stopping regions indicated by some of the nodes of the binomial tree ($M = 1000$); the exponential boundary curves of the tree are shown in black. The left-hand curve cuts the lower part of the stopping region. Only nodes "inside" the tree are shown; to save storage, nodes in the interior of the upper part are removed.

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* parameters taken from G.H. Meyer: Numerical investigation of early exercise in American puts with discrete dividends. J. Comput. Finance 5,2 (2002) 37–53.

Figure 2 uses the scaling $x := \frac{S}{K}$, and illustrates the surface $y(x) = V(Kx, t)$ (in red). The green line bounds the stopping area; compare Figure 1.

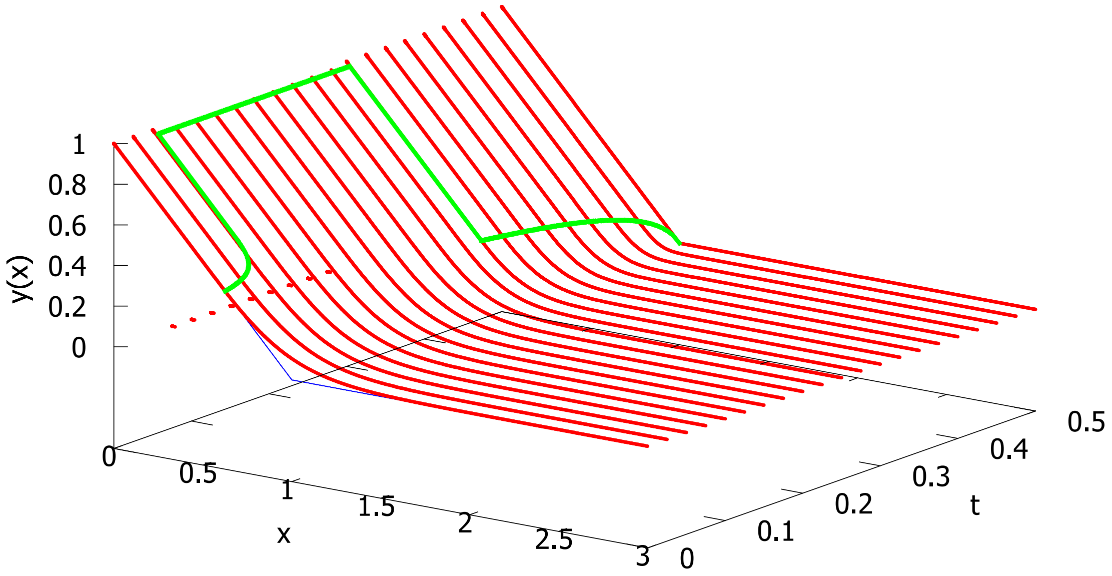

Figure 2: V over S and t, with S scaled to $x = \frac{S}{K}$.

In contrast, for a **proportional dividend** $D = \tilde{q}S$ there is a lower part of the stopping area extending to t_D, where the limiting early-exercise curve (green line in Figure 3) tends to $(S, t) = (0, t_D)$ with the slope

$$\lim_{t \to t_D,\, t < t_D} S_{\mathrm{f}}'(t) = -\frac{r}{\tilde{q}} \,.$$

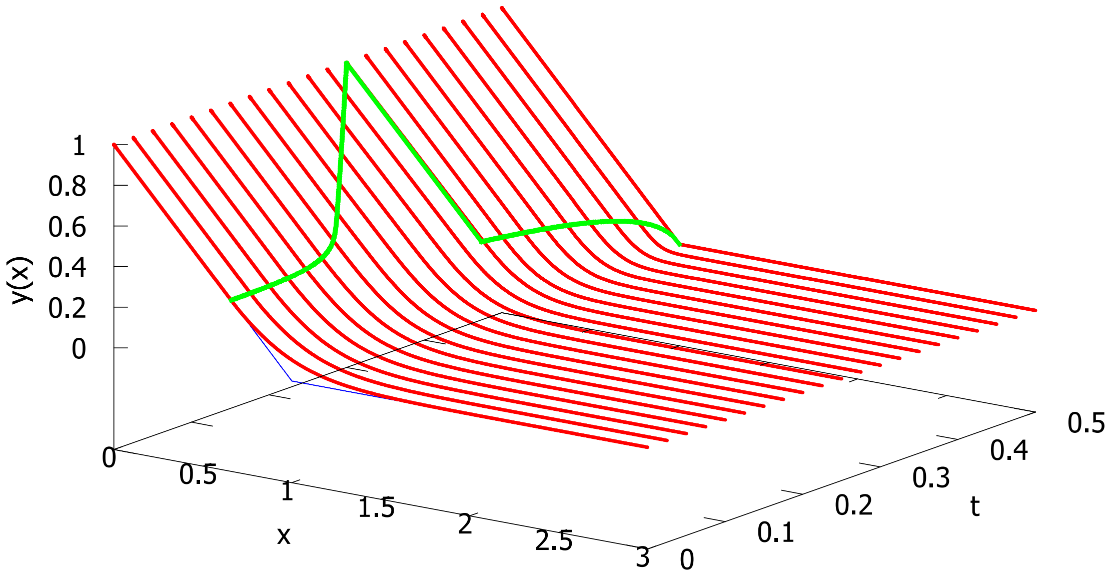

Figure 3: V over S and t, with $x = S/K$. Example as above, except for a proportional dividend payment $D = 0.02 \cdot S$

(Figures 2 and 3 with kind permission of Arthur Glados; calculated with a method of lines as in [Mey02]. For discrete dividends see also Topic 5.)