Preface

Fifteen years have elapsed after the second edition of *Practical Bifurcation and Stability Analysis* was published. During that time period the field of computational bifurcation has become mature. Today, bifurcation mechanisms are widely accepted as decisive phenomena for explaining and understanding stability and structural change. Along with the high level of sophistication that bifurcation analysis has reached, the research on basic computational bifurcation algorithms is essentially completed, at least in ordinary differential equations. The focus has been shifting from mathematical foundations towards applications.

The evolution *from equilibrium to chaos* has become commonplace and is no longer at the cutting edge of innovation. But the corresponding methods of *practical bifurcation and stability analysis* remain indispensable instruments in all applications of mathematics. This constant need for practical bifurcation and stability analysis has stimulated an effort to maintain this book on a present-day level. The author's endeavor has resulted in this third edition. It is based on more than three decades of practical experience with the subject, and on many courses given at several universities.

Like the previous editions, this third edition consists of three parts. In the first part (Chapters 1 to 3) an introduction into bifurcation and stability phenomena is given, basically restricted to models built of ordinary differential equations. Phenomena such as birth of limit cycles, hysteresis, or period doubling are explained. The second part (Chapters 4 to 7) introduces computational methods for analyzing bifurcation and stability. This includes continuation and branch switching as basic means. The final part (Chapters 8 and 9) gives qualitative insight that may help in understanding and assessing computational results. Such an interpretation of numerical results is based on singularity theory, catastrophe theory, and chaos theory.

This book emphasizes basic principles and shows the reader how the methods result from combining and, on occasion, modifying the underlying principles. The book is written to address the needs of scientists and engineers and to attract mathematicians. Mathematical formalism is kept to a minimum; the style is not technical, and is often motivating rather than proving. Compelling examples and geometrical interpretations are essential ingredients in the style. Exercises and projects complete the text. The book is self-contained, assuming only basic knowledge in calculus. The extensive bibliography includes many references on analytical and numerical methods, applications in science and engineering, and software. The references may serve as first steps in finding additional material for further research.

The book attempts to provide a practical guide for the performance of parameter studies.

New in This Edition

This third edition has been partly reorganized. The main change is a newly written Chapter 3. The third chapter of the second edition was removed, part of its contents was added to the fourth chapter. The new Chapter 3 is devoted to applications and extensions of standard ODE approaches. It includes brief expositions on delay differential equations, on differential-algebraic equations, and on pattern formation. This last aspect is concentrating on reaction-diffusion problems with applications in nerve models. Finally, this new third chapter addresses the aspect of deterministic risk, which can be tied to bifurcation. Applications include production of blood cells, dry friction, a flip-flop circuit, Turing bifurcation, and an electric power generator.

In addition to the new Chapter 3, several new sections have been inserted. In Chapter 5, the new Section 5.5 summarizes the information on second-order derivatives. In Chapter 7, on periodic orbits, the Section 7.6 on numerical aspects of bifurcation was enlarged. In Chapter 9, the section on fractal dimensions has been extended, and a new section has been added on the control of chaos, with focus on the OGY method.

Apart from these expanded sections, the entire book has been thoroughly reworked and revised. There are many new figures, while other figures have been improved. A considerable number of new references guide the reader to some more recent research or applications. The additions of this third edition are substantial; this may be quantified by the increase in the number of pages (+16%), figures (+19%), or references (+22%). The author has attempted to follow the now generally adopted practice to use *branching* and *bifurcation* as synonyms.

How to Use the Book

A path is outlined listing those sections that provide the general introduction into bifurcation and stability. Readers without urgent interest in computational aspects may wish to concentrate on the following:

Sections 1.1 to 1.4;
all of Chapter 2;
part of Chapter 3;
part of Section 5.4.2, and Sections 5.5, 5.6.4, and 5.6.5;
Section 6.1, example in Section 6.4, and Section 6.8;

Sections 7.1 to 7.4, 7.7, and 7.8;
all of Chapter 8; and
all of Chapter 9.

Additional information and less important remarks are set in small print. On first reading, the reader may skip these parts without harm. Readers with little mathematical background are encouraged to read Appendices 1 to 3 first. Solutions to several of the exercises are given later in the text. References are not meant as required reading, but are hints to help those readers interested in further study. The figures framed in boxes are immediate output of numerical software.

I hope that this book inspires readers to perform their own experimental studies. The many examples and figures should provide a basis and motivation to start right away.

Köln, September 2009 *Rüdiger Seydel*

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