Preface

Preface to the Sixth Edition

Five years after the fifth edition came out, there is a need to include additional results, and to improve explanations of methods and algorithms further. Besides numerous enhancements of small details there are new subjects.

New in the sixth edition are, for example, methods for smoothing in Chapter 1, and an introduction into basic aspects of efficiency. In Chapter 2, acceptance-rejection methods for generating random numbers are explained, with application to the Ziggurat algorithm for calculating normal variates. Chapter 3 on Monte Carlo methods now includes a subsection on positive solutions, and an outline of the antithetic variance reduction. Iterative approaches in Chapter 4 have become less important, in favor of direct methods.

To support the important role tree methods play in practice, an entire new appendix (Appendix D) is devoted these methods. This appendix includes trinomial methods, multidimensional trees, and implied trees for variable volatility. Further, how to handle discrete dividends is explained.

And the text is enriched by more figures. To facilitate understanding, many of the figures have been recalculated to become colored. Additional formalized exercises are included, and numerous hints at informal exercises are spread throughout the text.

On the technical side, the entire book has been transferred from *plain-TeX* to *LaTeX*. This has offered plenty of occasion to work the book over thoroughly. Additional colored figures can be found in the collection *Topics for Computational Finance* (shortly *Topics fCF*) on the internet platform `www.compfin.de`.

Köln, April 2017 Rüdiger U. Seydel

Preface to the Fifth Edition

Financial engineering and numerical computation are genuinely different disciplines. But in finance many computational methods are used and have become indispensable. This book explains how computational methods work in financial engineering. The main focus is on computational methods; financial engineering is the application. In this context, the numerical methods are *tools*, the tools for computational finance.

Faced with the vast and rapidly growing field of financial engineering, we need to choose a subarea to avoid overloading the textbook. We choose the attractive field of option pricing, a core task of financial engineering and risk analysis. The broad field of option pricing is both ambitious and diverse enough to call for a wide range of computational tools. Confining ourselves to option pricing enables a more coherent textbook and avoids being distracted away from computational issues. We trust that the focus on option-related methods is representative of, or least helpful for, the entire field of computational finance.

The book starts with an introductory Chapter 1, which collects financial and stochastic background. The remaining parts of the book are devoted to computational methods. Organizing computational methods, roughly speaking, leads to distinguish stochastic and deterministic approaches. By "stochastic methods" we mean computations based on random numbers, such as Monte Carlo simulation. Chapters 2 and 3 are devoted to such methods. In contrast, "deterministic methods" are frequently based on solving partial differential equations. This is discussed in Chapters 4, 5 and 6. In the computer, finally, everything is deterministic. The distinction between "stochastic" and "deterministic" is mainly to motivate and derive different approaches.

All of the computational methods must be adapted to the underlying model of a financial market. Here we meet different kinds of stochastic processes, from geometric Brownian motion to Lévy processes. Based on the chosen process an option model is selected. The classical choice is the Black–Scholes model for vanilla options with one underlying asset. This benchmark market model is "complete" in that all claims can be replicated. Established by Black, Merton, Scholes and others, this model is the main application of methods explained in Chapters 2 – 6. Chapter 7 goes beyond and addresses more general models. Allowing for jump processes, transaction costs, multi-asset underlyings, or more complicated payoffs, leads to incomplete markets. Computational methods for incomplete markets are briefly discussed in Chapter 7.

This book has been published in several editions. The first German edition (2000) was mainly absorbed by the Black–Scholes equation. Later editions (first English edition 2002) were carefully opened to more general models and a wider selection of methods. The book has grown with the development of the field. Faced with a large variety of possible computational tools, this book attempts to balance the need for a sufficient number of powerful algorithms with the limitations of a textbook. The balance has been gradually shifting over the years and editions. Numerous investigations in our research group have influenced the choice of covered topics. We have implemented and tested many dozens of algorithms, and gained insight and experience. A significant part of this knowledge has entered the book.

Readership

This book is written from the perspective of an applied mathematician. The level of mathematics is tailored to advanced undergraduate science and engineering majors. Apart from this basic knowledge the book is self-contained and can be used for a course on the subject. The intended readership is interdisciplinary and includes professionals in financial engineering, mathematicians and scientists of many other fields.

An expository style may attract a readership ranging from students to practitioners. Methods are introduced as tools for immediate application. Formulated and summarized as algorithms, a straightforward implementation in computer programs should be possible. In this way, the reader may learn by computational experiment. *Learning by calculating* will be a possible way to explore several aspects of the financial world. In some parts, this book provides an algorithmic introduction to computational finance. To keep the text readable for a wide audience, some aspects of proofs and derivations are exported to exercises at which hints are frequently given.

New in the Fifth Edition

The revisions to this fifth edition are much more extensive than those of previous editions. Compared to the fourth edition, the page count has increased by about 100 pages. The main addition is Chapter 7, which is devoted to incomplete markets. It begins with an introduction to nonlinear Black–Scholes type partial differential equations, as they arise from considering transaction costs or ranges for a stochastic volatility. Numerical approaches require instruments that converge to viscosity solutions. These solutions are introduced in an appendix. The role of monotonicity of numerical schemes is outlined. Lévy processes, with a focus on Merton's jump-diffusion and a numerical approach to the resulting partial integro-differential equation are then addressed. The chapter ends with an exposition on how the Fourier transform can be applied to option pricing. To complete the introduction of more general models and methods, the Dupire equation is outlined in a new appendix.

In addition to the new Chapter 7, several larger extensions and new Sections have been written for this edition. The calculation of Greeks is described in more detail, including the method of adjoints for a sensitivity analysis (new Section 3.7). Penalty methods are introduced and applied to a two-factor model in the new Section 6.7. More material is presented in the field of analytical methods; in particular, Kim's integral representation and its computation have been added to Chapter 4. Tentative guidelines on how to compare different algorithms and judge efficiency are given in the new Section 4.9. The chapter on finite elements has been extended with a discussion of two-asset options.

Apart from additional material listed above, the entire book has been thoroughly revised. The clarity of the expository parts has been improved; all sections have been tested in the class room. Numerous amendments, further figures, exercises and many references have been added. For example, the principal component analysis and its applications are included and the role of different boundary conditions is outlined in more detail.

How to Use this Textbook

Exercises are stated at the end of each chapter. They range from easy routine tasks to laborious projects. In addition to these explicitly formulated exercises, plenty of "hidden" exercises are spread throughout the book, with comments such as "the reader may check." Of course, the reader is encouraged to fill in those small intermediate steps that are excluded from the text.

This book explains the basic ideas of several approaches, presenting more material than is accomplishable in one semester. The following guidelines have proved successful in teaching:

First Course:

Chapter 1 without Section 1.6.2,
Chapter 2,
Chapter 3 without Section 3.7,
Chapter 4, with one analytic method out of Section 4.8,
  and without Section 4.9,
Chapter 6, or parts of it.

Second Course:

the remaining parts, in particular
Chapter 5 and Chapter 7.

Depending on the detail of explanation, the first course could be for undergraduate students. The second course may attract graduate students.

Extensions in the Internet

There is an accompanying internet page:

`www.compfin.de`

This is intended to serve the needs of the computational finance community and provides complementary material to this book. In particular, the collection *Topics in Computational Finance*, which is under construction, presents several of our findings or figures that would go beyond the limited scope of a textbook. In its final state, *Topics* is anticipated as a companion volume to the *Tools*.

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